

# Financial Stability and Basel II

by

Paul H. Kupiec\*

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## ABSTRACT

Basel II will enhance financial stability if it improves upon the 1988 Basel Accord's ability to ensure that systemically important institutions retain adequate minimum capital. Quantitative impact studies (QIS) show that the Advanced Internal Ratings Based (AIRB) approach—the Basel II approach that will be used by the largest internationally active banks—requires significantly less capital than the 1988 Basel Accord. QIS results show AIRB banks produce widely divergent estimates of minimum capital for substantially identical risks. These results raise questions about AIRB calibration and the regulations that define AIRB inputs. AIRB capital requirements are compared to capital set using a fully-specified equilibrium structural model of credit risk. The comparison shows that a fully compliant AIRB bank may produce a wide range of capital requirements for a given level of risk exposure owing to regulatory ambiguities regarding the measurement of loss given default and exposure at default. The results show the AIRB approach will undercapitalize portfolio credit risk relative to the Basel II target of 99.9 percent bank solvency, and capital shortfalls can be substantial. In contrast, the Foundation Internal Ratings Based (FIRB) approach allocates significantly more capital than necessary to achieve the supervisory objective. These results suggest the potential for increased systemic risk under Basel II, as regulations specify the weakest risk management standards for large complex AIRB banks.

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\* Division of Insurance and Research, Federal Deposit Insurance Corporation. The views expressed are those of the author and do not reflect the views of the FDIC. I am grateful to Sudipto Bhattacharya, Charles Calomiris, Sanjiv Das, Mark Flannery, Charles Goodhart, Michael Gordy, David Jones, George Kaufman, Robert Jarrow, Wenying Jiangli, Dilip Madan, Jack Reidhill, and Haluk Unal for comments on an earlier draft of this paper. Email: [pkupiec@fdic.gov](mailto:pkupiec@fdic.gov)

## Financial Stability and Basel II

### 1. INTRODUCTION

Under the June 2004 Basel II agreements, national supervisory authorities may choose among three alternative frameworks to set minimum regulatory capital for their internationally active banks.<sup>1</sup> One approach, the standardized approach, sets minimum capital standards using a modified version of the 1988 Basel Capital Accord that links capital requirements to external credit ratings. The remaining two approaches, the so-called Foundation (FIRB) and Advanced (AIRB) Internal Ratings Based approaches use a regulatory model to assign minimum capital requirements. The model assigns capital according to an individual credit's probability of default (*PD*), loss given default (*LGD*), maturity (*M*), and expected exposure at default (*EAD*).

The Basel II AIRB and FIRB frameworks set minimum regulatory capital requirements using a modified version of an industry model, the so-called Gaussian asymptotic single risk factor model of credit risk. This model, originally developed by Vasicek (1991) (hereafter, the Vasicek model), has been extended by many including Finger (1999), Schönbucher (2000) and Gordy (2003). The Vasicek model assumes that default risk is generated by Gaussian uncertainty and includes a single common source of risk and independent risk factors for each credit. Loss given default (*LGD*) is specified exogenously.

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<sup>1</sup> A revised version of the June 2004 Basel II agreement appears in Basel Committee for Banking Supervision (2006b). In the U.S., banking supervisors have determined that Basel II implementation will require only the largest banks, the so-called core banks, to adopt the AIRB approach, while other banks may petition supervisors for AIRB capital treatment (so-called opt-in banks). The remaining banks (so-called general banks) will continue using the capital requirements specified in the 1988 Basel Capital Accord. Core banks are defined as institutions with total banking (and thrift) assets of \$250 billion or more or total on-balance-sheet foreign exposure of \$10 billion or more. General banks likely will be subject to a modified version of the 1988 Basel Accord, so-called Basel 1A, but the potential modifications have to be finalized.

The model assumes bank portfolios are fully-diversified with respect to idiosyncratic sources of risk, and capital is needed to buffer the loss uncertainty associated with the single non-diversifiable source of risk.<sup>2</sup> Under the AIRB rule, banks must estimate each credit's *PD*, *EAD* and *LGD*. Under the FIRB rule, *LGD* is a fixed regulatory parameter. In contrast to the Vasicek model, IRB capital rules specify default correlation as deterministic function of *PD* that differs according to other characteristics of the credit.<sup>3</sup>

In designing Basel II, the Basel Committee on Banking Supervision (BCBS) specified a prudential standard of a 99.9 percent bank solvency rate of over a one-year horizon. The BCBS defend the framework's complexity and targeted prudential rigor as necessary given the complexity of large international banking organizations and the need to foreclose opportunities for regulatory arbitrage that exist under the 1988 Basel Accord.<sup>4</sup> Alternative Basel II approaches are calibrated so that the AIRB approach produces the lowest capital requirements to encourage banks to transition from the Standardized and FIRB approaches to the AIRB approach. Capital savings accorded under the AIRB are intended to offset the costs associated with developing and operating AIRB systems and to reflect efficiencies that are presumed to be generated by more efficient measurement of credit risk and assignment of minimum capital.

BCBS discussions of the Basel II suggest that the new capital standard should fortify the minimum bank capital requirements of internationally active banks. In contrast, the most recent Quantitative Impact Studies (QIS) rules show that large internationally active banks will benefit from large capital reductions under Basel II, especially under the AIRB approach. These QIS studies also show that AIRB estimates of minimum capital requirements for positions with similar risks may vary widely across banks. The decline and variability in regulatory capital estimates raises issues regarding the calibration of the AIRB

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<sup>2</sup> See Gordy (2003) for further discussion.

<sup>3</sup> The correlation function differs for corporate, sovereign and bank, small- and medium-sized enterprises, specialized lending, commercial real estate, residential mortgages, credit cards, and other retail exposures.

<sup>4</sup> See for example, BCBS (1998, June 1999), Jones (2000), or Myer (2001).

model and the regulations that guide *PD*, *EAD*, and *LGD* inputs into the AIRB rule. To date, no published study has analyzed the rigor of the prudential standards that are set by the June 2006 Basel II IRB models or examined the implications of their calibration and input specifications.

This paper analyzes the minimum solvency standards that are set under the June 2006 Basel II IRB approaches for corporate, sovereign, and bank credits.<sup>5</sup> Using a fully parameterized equilibrium structural model of credit risk, capital allocations are derived for credit portfolios that satisfy the assumptions that underpin the FIRB and AIRB models—default correlations are driven by a single common factor, and idiosyncratic risk is fully diversified. In a calibration exercise, the June 2006 IRB specifications of the AIRB and FIRB capital requirements are compared to capital estimates calculated from a full structural capital allocation model.

The calibration comparison shows that the AIRB approach substantially undercapitalizes credit risks, producing a capital shortfall that varies depending on the definitions of *EAD* and *LGD* that are used to calculate AIRB capital requirements. Under the definitions of *EAD* and *LGD* that are most likely to be used, bank default rates may exceed 5 percent when minimum capitalization rates are fully compliant with Basel II AIRB rules. In contrast, the FIRB will substantially over-capitalize credit risks relative to the 99.9 percent target solvency rate. Overcapitalization is a result of the FIRB assumption of 45 percent *LGD*, an assumption that overestimates the loss rates on the credits examined in this study.

The analysis identifies an important source of ambiguity in the Basel II capital rules. Under the current framework, permissible definitions for *EAD* and *LGD* can result in AIRB prudential standards that are far weaker than those set by the 1988 Basel Accord or those mandated under the alternative Basel II capital regimes. The AIRB rule, moreover, is subject to interpretation that may lead to substantial variation in bank estimates of the capital needed for a given credit risk exposure so all AIRB banks need not face the same prudential capital

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<sup>5</sup> The June 2006 AIRB and FIRB calibrations for corporate, sovereign and bank credits are unchanged from the calibration in the June 2004 Basel II agreements.

standard. The FIRB prudential standard, in contrast, is much more conservative than the AIRB standard that will apply to the largest systemically important banks.

Should AIRB bank capitalization levels approach the regulatory minimums allowed under Basel II, the largest internationally active banking institutions will operate under a solvency standard that increases the implicit public safety net subsidy. The “tilted playing field” established under Basel II may encourage banking system assets to migrate toward AIRB banks in order to maximize the value of implicit safety net subsidies that may accrue to large systemically-important banks. Ultimately the capital benefits that accrue to AIRB banks—benefits that are intended to encourage adoption of sophisticated risk management systems—may undermine the Basel II objective of buttressing the stability of the international financial system.

An outline of this paper follows. Section 2 supplies background on the development and calibration of Basel II. Section 3 summarizes the general methodology for constructing economic capital allocations. Section 4 derives capital allocations for portfolio credit risk in the context of an equilibrium structural model. Section 5 derives equilibrium credit risk capital rules under assumptions that mimic those that underlie the Basel II IRB models. Section 6 reviews the procedures for setting minimum capital requirements under the Basel IRB approaches. Section 7 discusses the calibration results and Section 8 concludes the paper.

## **2. BACKGROUND**

In designing its prudential standards, the Basel Committee on Banking Supervision (BCBS) expressed an objective that includes setting minimum regulatory capital requirements at a level consistent with a minimum bank solvency margin of 99.9 percent over a one-year horizon.<sup>6</sup> The rationale for setting this solvency standard has not been explained in official BCBS documents, and yet influential BCBS member studies have

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<sup>6</sup> Basel Committee on Banking Supervision, 2004, paragraph 667.

suggested that this solvency standard is approximately consistent with the standard set by the 1988 Basel Accord.<sup>7</sup>

The BCBS arrived at the June 2004 IRB framework through a process of industry consultation and international compromise. There is no BCBS document that provides formal back testing evidence of the solvency standards that are achieved under the alternative Basel II regimes. Instead, a series of technical working papers and an early draft of the proposed framework were released by the BCBS for public comment. A series of three Quantitative Impact Studies (QIS) studies required banks to estimate the effects of alternative IRB calibrations on their minimum regulatory capital requirements. The June 2004 calibrations reflect an iterative process in which consecutive IRB formulations were modified following industry commentary, internal BCBS discussions and negotiations, and review of the QIS results. Calibrations were modified with a goal of achieving capital neutrality while creating incentives that encouraged banks to adopt the IRB approaches.<sup>8</sup>

Following the June 2004 publication of the Basel II framework, two additional QIS studies have been conducted: QIS 4 in the United States, Germany and South Africa, and QIS 5 in adopting countries in the remainder of the world. Both studies reported substantial declines in minimum capital requirements for AIRB banks relative to required capital under the 1988 Basel Accord.

The Spring 2005 QIS 4 study included 26 U.S. institutions, all of which reported using the AIRB approach.<sup>9</sup> The results show that, in aggregate, minimum regulatory capital for these institutions fell by 15.5 percent under the AIRB. Among these banks, the median reduction in capital was 26 percent and the median reduction in Tier I capital requirements was 31 percent. Of the few banks that experienced increases in minimum capital requirements under the AIRB, the increases were driven primarily by increases in capital for consumer retail portfolios and to a lesser extent by equity exposures.

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<sup>7</sup> See for example Jackson, Perraudin, and Saporta (2002).

<sup>8</sup> The Basel Committee on Banking Supervision, October 2002, paragraphs 46-47.

<sup>9</sup> See the Federal Reserve Board Press release, "Summary Findings of the Fourth Quantitative Impact Study," available at [www.federalreserve.gov](http://www.federalreserve.gov)

In addition to large declines in capital, QIS 4 results show a high degree of dispersion in reported estimates of minimum capital requirements. Banks reported widely divergent capital estimates for their constituent portfolios (corporate, SME, etc.). While these differences could owe to true difference in bank risk profiles as a result of differentiation among customer bases and business strategies, additional analysis using shared national credit data indicated that banks reported widely divergent capital estimates for positions with substantially similar risk characteristics. Further analysis suggests that a significant share of the variation in QIS 4 results may be attributed differences in bank estimates of *PDs* and *LGDs* among credits with approximately equivalent risk characteristics. For the corporate, sovereign, and bank credit portfolio, for example, QIS 4 *LGD* estimates on non defaulted credits varied from about 15 to 55 percent across banking institutions.

The Spring 2006 QIS 5 study included 382 banks in 32 countries outside of the U.S..<sup>10</sup> Of the banks that participated, the largest internationally active banks, so-called Group 1 banks, posted capital declines of 7.1 percent on average under the AIRB approach. Smaller banks, so called Group 2 banks, primarily nationally focused institutions, posted much larger declines in minimum regulatory capital.<sup>11</sup> Within Europe,<sup>12</sup> Group 1 banks posted average capital declines of 8.3 percent under the AIRB. For European Group 2 banks, declines averaged 26.6 percent under the AIRB. Of the banks that experienced large declines in minimum regulatory capital requirements, the declines were attributed to bank concentrations in retail lending, especially residential mortgages. The BCBS summary of QIS 5 results does not provide a very detailed analysis of the dispersion of bank minimum capital estimates. The study does however report significant variation in AIRB input values.

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<sup>10</sup> See, BCBS (2006a). QIS 5 AIRB capital rules include a 1.06 scaling factor that was not included in the June 2004 calibration or the instructions that guided QIS 4. The inclusion of this scaling factor means the reported capital declines will appear less severe than those reported in the U.S..

<sup>11</sup> BCBS (2006a).

<sup>12</sup> So-called CEBS (Committee of European Bank Supervisors) banks.

*LGD* estimates for wholesale credits, for example, range from 10.8 to 67.6 percent across reporting banks.

### 3. REGULATORY CAPITAL FOR CREDIT RISKS

Basel II is designed to ensure a one-year bank solvency margin of 99.9 percent; that is, Basel II minimum capital requirements ostensibly are formulated to ensure that over a one-year horizon, the probability that a compliant bank defaults on its financial obligations is less than 0.001 percent.

To model the Basel II capital constraint, it is assumed that capital allocations are set with a goal of maximizing leverage subject to ensuring that the bank is able to meet all the associated interest and principal payments with a minimum probability of  $\alpha = 99.9$  percent. To maximize leverage, the bank is assumed to issue a single class of discount funding debt that matures at time  $T = 1$  year.  $\alpha = 99.9$  percent is the bank's target solvency rate;  $1 - \alpha = 0.001$  percent is the bank's *ex ante* target probability of default.

The one-year solvency standard implies a capital allocation horizon of one year. The purchased asset  $A$ , has an initial market value  $A_0$ , and a time 1 random value of  $\tilde{A}_1$  with a cumulative density function  $\Psi(\tilde{A}_1, A_1)$ , and a probability density function  $\psi(\tilde{A}_1, A_1)$ .<sup>13</sup> Let  $\Psi^{-1}(\tilde{A}_1, 1 - \alpha)$  represent the inverse of the cumulative density function of  $\tilde{A}_1$  evaluated at  $1 - \alpha$ ,  $\alpha \in [0, 1]$ . Define an  $\alpha$  coverage VaR measure,  $VaR(\alpha)$ , as,

$$VaR(\alpha) = A_0 - \Psi^{-1}(\tilde{A}_1, 1 - \alpha) \quad (1)$$

$VaR(\alpha)$  is the loss amount that is exceeded by at most  $(1 - \alpha)$  of all potential future value realizations of  $\tilde{A}_1$ . Expression (1) measures value-at-risk relative to the initial market value

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<sup>13</sup> The construction of an optimal economic capital allocation is simplified when portfolios are composed of assets with non-negative market values. For purposes of this analysis, portfolio composition is restricted to include only long positions in fixed income claims that may generate losses that are bounded above by the initial market value of the credit. See Kupiec (2004a) for further discussion.

of the asset. When credit risk losses are bounded above by  $A_0$ ,  $\Psi^{-1}(\tilde{A}_1, 1-\alpha)$  is bounded below by 0.

Consider a capital allocation rule that sets equity capital equal to  $VaR(\alpha)$ . By definition, the probability the bank will experience a loss in excess of its initial equity value is at most  $100(1-\alpha)$  percent. Under the  $VaR(\alpha)$  capital allocation rule, the bank must borrow  $A_0 - VaR(\alpha)$  to finance the investment. If the bank borrows  $A_0 - VaR(\alpha)$ , it must promise to pay back *more* than  $A_0 - VaR(\alpha)$  if equilibrium interest rates and credit risk compensation are positive. Because the  $VaR(\alpha)$  capital allocation rule ignores time and the equilibrium returns that are required by bank creditors, the probability that the bank will default on its funding debt is greater than  $1-\alpha$  if the bank's debts can only be satisfied by raising funds through the sale of  $\tilde{A}_1$  at time  $T$ .

The capital allocation rule that meets the Basel II regulatory objective is: set equity capital equal to  $VaR(\alpha)$  plus the interest that will accrue on the bank's borrowings. An equivalent allocation is achieved by setting the par (maturity) value of the funding debt equal to  $VaR(\alpha)$  and estimating the funding debt's market value at issuance. The difference between the market value of the purchased asset and the proceeds from the funding debt issue is the equity capital needed to fund the investment and satisfy the solvency rate target. This capital allocation rule generalizes to a portfolio context.

#### **4. BASEL II CAPITAL REQUIREMENTS IN AN EQUILIBRIUM STRUCTURAL MODEL**

Estimation of the equilibrium interest cost on funding debt requires the use of formal asset pricing models or an empirical approximation to value a bank's funding debt. In this analysis, we will adopt the asset pricing approach. If the risk-free term structure is flat and a firm issues only pure discount bond, and asset values follow geometric Brownian motion,

under simplifying assumptions,<sup>14</sup> Black and Scholes (1973) and Merton (1974) (hereafter BSM) establish that the market value of a firm's debt is equal to the discounted value of the bond's par value (at the risk free rate), less the market value of a Black-Scholes put option written on the firm's assets. The put option has a maturity identical to the bond's maturity, and a strike price equal to the par value of the bond. The BSM model can be modified to produce capital allocation rules that are fully consistent with equilibrium conditions and the underlying structural features that determine the market value of bank debt and equity securities.

To simplify the discussion, assume all issued and purchased debt claims mature in one year. The assumption that bank-issued debt matures in one year is a convenience that avoids the need to mark-to-market bank funding liabilities at the end of the one-year regulatory horizon. The assumption that all credits mature in one year is consistent with default mode models of credit risk in which credits are modeled as either fully performing or defaulting at the end of the horizon of interest. The Vasicek model, and the Basel II IRB models that are derived from the Vasicek model are default mode models.

The BSM capital allocation framework can easily accommodate longer maturity claims purchased by the bank within the regulatory horizon of interest, but there is a cost in terms of mathematical complexity that is unnecessary for purposes of this paper. This so-called market-to-market capital allocation process, where long maturity credits are valued at the end of the capital allocation horizon, is discussed in Kupiec (2004a, 2004b).

Consider a bank whose only asset is a risky BSM discount bond issued by an unrelated counterparty that matures at date 1. Let  $B_0$  represent the initial market value of this bond. Assume that the bank will fund this bond with its own 1-year discount debt issue, and equity. In this setting, the bank's funding debt issue is a compound option. Let  $\tilde{A}_1$  and

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<sup>14</sup> There are no taxes, transactions are costless, short sales are possible, trading takes place continuously, if borrowers and savers have access to the debt market on identical risk-adjusted terms, and investors in asset markets act as perfect competitors.

$Par_p$  represent, respectively, the time 1 value of the assets that support the discount debt investment and the par value of the purchased bond. Let  $Par_F$  represent the par value of the discount bond that is issued by the bank to fund the investment. The end-of-period cash flows that accrue to the bank's debt holders are,

$$Min\left[Min\left(\tilde{A}_1, Par_p\right), Par_F\right]. \quad (2)$$

The initial equilibrium market value of the bank's discount bond issue is the discounted (at the risk free rate) expected value of the end-of-period funding debt cash flows taken with respect to the equivalent martingale probability distribution for the assets,  $\tilde{A}_1^q$ ,

$$E\left[Min\left[Min\left(\tilde{A}_1^q, Par_p\right), Par_F\right]\right]e^{-r_f} \quad (3)$$

where

$$\tilde{A}_1^q \sim A_0 e^{\left(r_f - \frac{\sigma^2}{2}\right) + \sigma \tilde{z}} \quad (4)$$

and  $\tilde{z}$  is a standard normal random variable.

The payoff of the bank's purchased bond is given by  $Min\left[Par_p, \tilde{A}_1\right]$ , where  $\tilde{A}_1$  is the

asset value at date 1,  $\tilde{A}_1 \sim A_0 e^{\left(\mu - \frac{\sigma^2}{2}\right) + \sigma \tilde{z}}$ , and  $\mu = r_f + \lambda\sigma$ , where  $\lambda$  is the market price of risk. Let  $\Phi(x)$  represent the cumulative standard normal distribution function evaluated at  $x$ , and let  $\Phi^{-1}(\alpha)$  represent the inverse of this function for  $\alpha \in [0,1]$ . Because  $\tilde{A}_1$  is monotonic in  $\tilde{z}$ , the upper bound on the par (maturity) value of the funding debt that can be issued under the target solvency constraint is,

$$Par_F(\alpha) = \Psi^{-1}\left(\tilde{A}_1, 1 - \alpha\right) = A_0 e^{\left[\mu - \frac{\sigma^2}{2}\right] + \sigma \Phi^{-1}(1 - \alpha)}. \quad (5)$$

The initial market value of this funding debt issue is,  $B_{F0}(\alpha)$ ,

$$B_{F0}(\alpha) = E\left[Min\left[Min\left(\tilde{A}_1^q, Par_p\right), Par_F(\alpha)\right]\right]e^{-r_f}. \quad (6)$$

and the initial equity allocation consistent with the target solvency rate  $\alpha$ ,  $E(\alpha)$ , is,

$$E(\alpha) = B_0 - E\left[Min\left[Min\left(\tilde{A}_1^q, Par_p\right), Par_F(\alpha)\right]\right]e^{-r_f}. \quad (7)$$

In the single asset case, when the probability of default on the purchased bond is less than or equal to  $(1 - \alpha)$ , the bond can be financed 100 percent with bank debt without violating the solvency constraint ( $Par_F(\alpha) = Par_p$ ). When the probability of default on the purchased bond exceeds  $(1 - \alpha)$ , capital is required, and  $Par_F(\alpha) < Par_p$ . In this case, expression (7) implies a dollar capital allocation,

$$E(\alpha) = B_0 - A_c e^{r_f} \Phi \left( \frac{\ln(A_c) - \ln(A_0) - \left( r_f - \frac{\sigma^2}{2} \right)}{\sigma} \right) - A_0 \Phi \left( \frac{\ln(A_c) - \ln(A_0) - \left( r_f + \frac{\sigma^2}{2} \right)}{\sigma} \right) \quad (8)$$

where  $A_c = Par_F(\alpha) = A_0 e^{\left[ \mu - \frac{\sigma^2}{2} \right] + \sigma \Phi^{-1}(1-\alpha)}$ .

### ***Portfolio Capital***

Except for the need to derive a probability density function for the portfolio's future value distribution under both the physical and the equivalent martingale measures, the process for setting portfolio capital requirements mirrors the calculations for a single asset. In most cases, credit portfolios do not have density functions that admit a closed-form expression for either the par value of the funding debt or its initial market value. Monte Carlo simulation is often required to estimate  $VaR(\alpha)$  and the par value of the funding debt, and pricing the funding debt may require numerical evaluation of a high order integral. The next section considers portfolio capital allocation under BSM assumptions when asset price dynamics are generated by a single common factor and idiosyncratic risk is fully diversified. These assumptions reduce significantly the complexity of portfolio capital calculations.

## **5. BASEL II CAPITAL UNDER ASYMPTOTIC SINGLE FACTOR ASSUMPTIONS**

Capital allocation calculations are simplified if a portfolio is well-diversified and asset values are driven by a single common factor in addition to individual idiosyncratic factors. Let  $dW_M$  represents a standard Wiener process common in all asset price dynamics,

and  $dW_i$  represents an independent standard Weiner process idiosyncratic to the price dynamics of asset  $i$ . Assume that asset price dynamics for firm  $i$  are given by,

$$dA_i = \mu A_i dt + \sigma_M A_i dW_M + \sigma_i A_i dW_i, \quad (9)$$

$$dW_i dW_j = \rho_{ij} = 0, \quad \forall i, j.$$

$$dW_i dW_M = \rho_{im} = 0, \quad \forall i.$$

Under these dynamics, asset prices are log normally distributed,

$$\tilde{A}_{iT} = A_{i0} e^{\left[ r_f + \lambda \sigma_M - \frac{1}{2}(\sigma_M^2 + \sigma_i^2) \right] T + (\sigma_M \tilde{z}_M + \sigma_i \tilde{z}_i) \sqrt{T}}, \quad (10)$$

where  $\tilde{z}_M$  and  $\tilde{z}_i$  are independent standard normal random variables. Under the equivalent martingale change of measure, asset values at time  $T$  are distributed,

$$\tilde{A}_{iT}^q = A_{i0} e^{\left[ r_f - \frac{1}{2}(\sigma_M^2 + \sigma_i^2) \right] T + (\sigma_M \tilde{z}_M + \sigma_i \tilde{z}_i) \sqrt{T}}. \quad (11)$$

Under these price dynamics, the correlation between geometric asset returns is,

$$\text{Corr} \left[ \frac{1}{T} \ln \left( \frac{\tilde{A}_{it}}{A_{i0}} \right), \frac{1}{T} \ln \left( \frac{\tilde{A}_{jt}}{A_{j0}} \right) \right] = \frac{\sigma_M^2}{(\sigma_M^2 + \sigma_i^2)^{\frac{1}{2}} (\sigma_M^2 + \sigma_j^2)^{\frac{1}{2}}}, \quad \forall i, j. \quad (12)$$

If the model is further specialized so that the volatilities of assets' idiosyncratic factors are assumed identical,  $\sigma_i = \sigma_j = \bar{\sigma}, \forall i, j$ , the pair-wise asset return correlations are,

$$\rho = \text{Corr} \left[ \frac{1}{T} \ln \left( \frac{\tilde{A}_{it}}{A_{i0}} \right), \frac{1}{T} \ln \left( \frac{\tilde{A}_{jt}}{A_{j0}} \right) \right] = \frac{\sigma_M^2}{\sigma_M^2 + \bar{\sigma}^2} \quad \forall i, j. \quad (13)$$

### ***Asset Return Distributions***

The  $T$ -period return on BSM risky bond  $i$  that is held to its maturity date  $T$  is,

$$\tilde{M}_{iT} = \frac{1}{B_{i0}} \left( \text{Min}(\tilde{A}_{iT}, \text{Par}_i) \right) - 1. \quad (14)$$

$\tilde{M}_{iT}$  is bounded in the interval  $[-1, a]$ , where  $a$  is a finite constant. When return realizations are in the range,  $-1 < M_{iT} < 0$ ,  $M_{iT}$  represents the loss rate on the bond held to maturity.

For realizations in the range,  $0 < M_{iT} < \frac{\text{Par}_i}{B_{i0}} - 1$ , the bond has defaulted on its promised

payment terms, but the bond has still generated a positive return. A fully performing bond

posts a return equal to  $\frac{\text{Par}_i}{B_{i0}} - 1 < a$  which is finite by assumption (and in practice).

A bond's physical return distribution (17) has an associated equivalent martingale return distribution,

$$\tilde{M}_{iT}^q = \frac{1}{B_{i0}} \left( \text{Min}(\tilde{A}_{iT}^q, \text{Par}_i) \right) - 1. \quad (15)$$

By construction, expressions (14) and (15) have identical support.

### ***Asymptotic Portfolio Return Distribution***

The  $T$ -period return on a portfolio of  $n$  risky individual credits,  ${}_P\tilde{M}_T$ , is

$${}_P\tilde{M}_T \equiv \frac{\sum_{i=1}^n \tilde{M}_{iT} B_{i0}}{\sum_{i=1}^n B_{i0}} \quad (16)$$

Let  $\left( {}_P\tilde{M}_T \mid \tilde{z}_M = z_M \right) = {}_P\tilde{M}_T \mid z_M$  represent the portfolio return conditional on a realization

of the common market factor,  $\tilde{z}_m = z_M$ ,

$${}_P\tilde{M}_T \Big| z_M = \frac{\sum_{i=1}^n \tilde{M}_{iT} \Big| z_M \cdot B_{i0}}{\sum_{i=1}^n B_{i0}} \quad (17)$$

If  $\psi(\tilde{M}_{iT} \Big| z_M)$  represents the conditional return density function, under the single common factor assumption for asset price dynamics,  $\psi(\tilde{M}_{iT} \Big| z_M)$  and  $\psi(\tilde{M}_{jT} \Big| z_M)$  are independent for  $\forall i \neq j$ .<sup>15</sup>

Consider a portfolio composed of equal investments in individual bonds that share identical *ex ante* credit risk profiles. That is, assume that the bonds in the portfolio are identical regarding par value  $\{Par_i = Par_j, \forall i, j\}$ , maturity  $\{T\}$ , and volatility characteristics,  $\{\sigma_i = \sigma_j = \bar{\sigma}, \forall i, j\}$ . Conditional bond returns are independent and identically distributed with a finite mean. As the number of bonds in the portfolio,  $N$ , grows without bound, the Strong Law of Large Numbers requires,

$$\lim_{n \rightarrow \infty} [{}_P\tilde{M}_T \Big| z_M] = \lim_{n \rightarrow \infty} \left[ \frac{\sum_{i=1}^n \tilde{M}_{iT} \Big| z_M}{n} \right] \xrightarrow{a.s.} E \left[ \psi(\tilde{M}_{iT} \Big| z_M) \right] \quad \forall z_M \quad (18)$$

The notation *a.s.* indicates “almost sure” convergence (convergence with probability one).

Under the BSM single factor assumptions, expression (18) becomes,

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<sup>15</sup> Independence in this non-Gaussian setting requires that an observation of the return to bond  $j$  be uninformative regarding the conditional distribution function for bond  $i$ ,  $\Pr(\tilde{M}_{iT} \Big| z_M) < a = \Pr(\tilde{M}_{iT} \Big| z_M) < a$  given that  $\tilde{M}_{jT} = M_{jT}$ ,  $\forall a, i \neq j$ . This condition is satisfied under the single common factor model assumption.

$$\begin{aligned} \lim_{n \rightarrow \infty} \left[ {}_P \tilde{M}_T \mid z_M \right] &= \frac{Par_i}{B_{i0}} \left[ 1 - \Phi \left( w_{iT} (z_M) \right) \right] \\ &+ \frac{Q(z_M)}{B_{i0}} \left[ 1 - \Phi \left( -w_{iT} (z_M) + \gamma_{iT} \right) \right] - 1 \end{aligned} \quad (19)$$

where,

$$\mu_{iT} (z_M) = \ln[A_{i0}] + \left[ r_f + \lambda \sigma_M - \frac{1}{2} (\sigma_M^2 + \sigma_i^2) \right] T + z_M \sigma_M \sqrt{T}$$

$$\gamma_{iT} = \sigma_i \sqrt{T}$$

$$w_{iT} (z_M) = \frac{\ln[Par_i] - \mu_{iT} (z_M)}{\gamma_{iT}}$$

$$Q(z_M) = e^{\mu_{iT} (z_M) + \frac{\gamma_{iT}^2}{2}}$$

Similarly, the equivalent martingale portfolio conditional return distribution is given by,

$$\begin{aligned} \lim_{n \rightarrow \infty} \left[ {}_P \tilde{M}_T^q \mid z_M \right] &= \frac{Par_i}{B_{i0}} \left[ 1 - \Phi \left( w_{iT}^q (z_M) \right) \right] \\ &+ \frac{Q^q(z_M)}{B_{i0}} \left[ 1 - \Phi \left( -w_{iT}^q (z_M) + \gamma_{iT} \right) \right] - 1 \end{aligned} \quad (20)$$

where,

$$\mu_{iT}^q (z_M) = \ln[A_{i0}] + \left[ r_f - \frac{1}{2} (\sigma_M^2 + \sigma_i^2) \right] T + z_M \sigma_M \sqrt{T}$$

$$\gamma_{iT} = \sigma_i \sqrt{T}$$

$$w_{iT}^q (z_M) = \frac{\ln[Par_i] - \mu_{iT}^q (z_M)}{\gamma_{iT}}$$

$$Q^q(z_M) = e^{\mu_{iT}^q(z_M) + \frac{\gamma_{iT}^2}{2}}$$

### ***Optimal Portfolio Capital Allocation in a Default Mode Approach***

In the default mode approach used in Basel II, credits are assumed to be held to maturity and either they fully perform or default. The hold-to-maturity assumption removes the need to mark portfolio credits to market at a date prior to maturity. In the approach that follows, it is assumed that all credits have an identical maturity of  $T = 1$  years consistent with the Basel II framework.

The portfolio return distribution is monotonic in  $z_M$ , so the capital allocation calculations need only involve the conditional portfolio return distributions. When expressed as a proportion of the investment portfolio's initial market value, the optimal par value of funding debt,  $par_F^P(\alpha) = \Psi^{-1}(\tilde{M}_1 + 1, 1 - \alpha)$  can be determined by setting  $z_M = \Phi^{-1}(1 - \alpha)$  and using expression (19) to solve for the end-of-horizon portfolio critical value,

$$par_F^P(\alpha) = \left( \begin{array}{l} \frac{Par_i}{B_{i0}} \left[ 1 - \Phi(w_{i1}(\Phi^{-1}(1 - \alpha))) \right] \\ + \frac{Q(z_M)}{B_{i0}} \left[ 1 - \Phi(-w_{i1}(\Phi^{-1}(1 - \alpha)) + \gamma_{i1}) \right] \end{array} \right) \quad (21)$$

To determine the market value of the funding debt, it is necessary to solve for the value of  $z_M$  that determines the default threshold under the risk neutral measure,  $\hat{z}_M$ ,

$$\hat{z}_M = \Phi^{-1}(1 - \alpha) + \lambda \quad (22)$$

$\hat{z}_M$  is one of the limits of integration needed to calculate the expected discounted payoff of the funding debt using the risk neutral measure. Expressed as a proportion of the investment portfolio's initial market value, the initial market value of the funding issue,  $b_{F0}^P(\alpha)$ , is ,

$$b_{F0}^P(\alpha) = e^{-r_f} \left( \int_{-\infty}^{\hat{z}_M} \left[ \frac{Par_i}{B_{i0}} [1 - \Phi(w_{i1}^q(z_M))] \right] \phi(z_M) dz_M \right. \\ \left. + \int_{-\infty}^{\hat{z}_M} \left[ \frac{Q^q(z_M)}{B_{i0}} [1 - \Phi(-w_{i1}^q(z_M) + \gamma_{i1})] \right] \phi(z_M) dz_M \right. \\ \left. + par_F^P(\alpha) [1 - \Phi(\hat{z}_M)] \right) \quad (23)$$

The economic capital allocation for the portfolio, expressed as a proportion of the portfolio's initial market value,  $K_{BSM}^P(\alpha)$  is,

$$K_{BSM}^P(\alpha) = 1 - e^{-r_f} \left( \int_{-\infty}^{\hat{z}_M} \left[ \frac{Par_i}{B_{i0}} [1 - \Phi(w_{i1}^q(z_M))] \right] \phi(z_M) dz_M \right. \\ \left. + \int_{-\infty}^{\hat{z}_M} \left[ \frac{Q^q(z_M)}{B_{i0}} [1 - \Phi(-w_{i1}^q(z_M) + \gamma_{i1})] \right] \phi(z_M) dz_M \right. \\ \left. + par_F^P(\alpha) [1 - \Phi(\hat{z}_M)] \right) \quad (24)$$

The dollar value capital required is  $\sum_{i=1}^n B_{i0} K_{BSM}^P(\alpha)$ .

Idiosyncratic risk is fully diversified, so when an additional credit is added to the portfolio, the marginal capital required to maintain the target solvency margin is equal to the portfolio's average capitalization rate (expression (24)) multiplied by the market value of the marginal credit added to the portfolio. Expression (24) represents the minimum regulatory capitalization rate for both the average and the marginal credit in an asymptotic portfolio

when credit risks are priced to satisfy BSM equilibrium conditions and capital is set to achieve a 99.9 percent solvency rate.

## 6. MINIMUM CAPITAL REQUIREMENTS UNDER THE BASEL II IRB APPROACHES

The June 2006 formula for calculating AIRB capital requirements for corporate, sovereign and bank exposures are  $EAD \cdot K$ , where  $K$ , is given by<sup>16</sup>

$$K = \left[ LGD \times \Phi \left[ \frac{1}{\sqrt{1-R}} \Phi^{-1}(PD) + \sqrt{\frac{R}{1-R}} \Phi^{-1}(.999) \right] - PD \times LGD \right] \left( \frac{1 + (M - 2.5)b}{1 - 1.5b} \right) \quad (25)$$

where,  $R = 0.12 \left( \frac{1 - e^{-50PD}}{1 - e^{-50}} \right) + 0.24 \left( 1 - \frac{1 - e^{-50PD}}{1 - e^{-50}} \right)$ ,  $b = (0.11852 - .05478 \ln(PD))^2$ ,

$PD$  is a credit's probability of default expressed as a percentage,  $LGD$  is a credit's expected loss given default expressed as a percentage,  $M$  is the credit's maturity in measured in years, and  $K$  represents the percentage capital requirement per dollar of  $EAD$  exposure. When

$M = 1$ , the maturity adjustment,  $\frac{1 + (M - 2.5)b}{1 - 1.5b} = 1$ . If for any credit,  $K < 0$ , regulatory

capital requirements are set to zero. FIRB capital requirements are calculated by using the AIRB capital requirement formula with  $LGD$  set at 45 percent.

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<sup>16</sup> BCBS (June 2006b), page 64.

## ***Basel II IRB Model Inputs***

### ***Probability of default (PD)***

For corporate and bank exposures, *PD* is the greater of the one-year *PD* or 0.03%. Basel II specifies that *PD* must be estimated using at least 5 years of data but for purposes of this analysis, *PD* is well defined by the BSM asset price dynamics and data estimation requirements are not an issue.

### ***Exposure at default (EAD)***

According to paragraph 474 of BCBS (June 2006b)

“*EAD* for an on-balance sheet or off-balance sheet item is defined as the expected gross exposure of the facility upon default of the obligor. For on-balance sheet items, banks must estimate *EAD* at no less than the current drawn amount, subject to recognizing the effects of on-balance sheet netting as specified in the foundation approach.”

The U.S. Basel II NPR (p.123) states, “*EAD* for the on-balance sheet component of a wholesale or retail exposure means (i) the bank’s carrying value for the exposure (including accrued but unpaid interest and fees)...” Thus, for simple loans or bonds without any additional attached line of credit, Basel II requires that *EAD* must be at least as large as the current carrying value of the asset at the time that minimum regulatory capital is calculated. For simple loan or bond positions, Basel II regulations do not include any further discussion; in particular, there is no discussion or guidance that suggests that *EAD* must be higher than the current carrying value of a simple fully drawn loan.

### ***Loss given default (LGD)***

Paragraph 297 of BCBS (June 2006b) requires that, “*LGD* be measured as... a percentage of the *EAD*.” Paragraph 468 discusses minimum requirements for AIRB bank *LGD* treatment:

“A bank must estimate an *LGD* for each facility that aims to reflect economic downturn conditions where necessary to capture the relevant risks. This *LGD* cannot be less than the long-run default-weighted average loss rate given default calculated based on the average economic loss of all observed defaults within the data source for that type of facility.”

The Basel II guidelines also do not set a lower bound on the *LGDs* that banks can use in the AIRB approach for corporate credits.

***Maturity (M)***

BCBS (June 2006b) paragraph 320 defines *M* as, “the greater of one year and the remaining effective maturity in years,” where the remaining effective maturity is the time

weighted-average of the instrument’s cash flows,  $Min \left( \frac{\sum_{\forall t} t \cdot CF_t}{\sum_{\forall t} CF_t}, 5 \text{ years} \right)$ , where  $CF_t$  is the

instrument cash flow *t* periods into the future, where *t* is measured in years.

Table 1: Calibration Assumptions

risk free rate	$r_f = .05$
market price of risk	$\lambda = .10$
market factor volatility	$\sigma_M = .10$
Firm specific volatility	$\bar{\sigma}_i = .20$
Initial market value of assets	$A_0 = 100$
correlation between asset returns	$\rho = .20$

## 7. IMPLIED BANK DEFAULT RISK UNDER THE BASEL II IRB APPROACHES

The economic capital allocations prescribed by the BSM model are compared to the minimum regulatory capital requirements that are set by the Basle II IRB approaches. Portfolio capital requirements are calculated using the BSM model and the Basel IRB approaches for portfolios with a wide range of risk characteristics. The assumptions regarding asset price dynamics that are maintained throughout the analysis appear in Table 1. All individual credits are assumed to have identical firm specific risk factor volatilities of 20 percent. The common factor has a volatility of 10 percent and the market price of risk is set at 10 percent. The risk free rate is 5 percent. The market and firm specific factor volatilities imply an underlying geometric asset return correlation of 20 percent.<sup>17</sup>

All credits in these asymptotic portfolios are assumed to have the same initial value, and all share an identical *ex ante* credit risk profile that is determined by the par value of the credit. The par values of individual credits are altered to change the credit risk characteristics of a portfolio. Consistent with Basel II requirements, the analysis focuses on a one-year capital allocation horizon and one-year maturity credits. In the comparison, the Basel II minimum capital calculations include expected loss since there is no scope for loan loss reserves in this one-period setting. The modified Basel II definition of  $K$  is,

$$K = \left[ LGD \times \Phi \left[ \frac{1}{\sqrt{1-R}} \Phi^{-1}(PD) + \sqrt{\frac{R}{1-R}} \Phi^{-1}(.999) \right] \right] \left( \frac{1 + (M - 2.5)b}{1 - 1.5b} \right) \quad (26)$$

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<sup>17</sup> When the bond  $PD$ s in Table 2 are input into the A-IRB approach, the correlation parameter,  $R$ , ranges from 13.6 (par value 70) to 22.7 percent (par value 55).

### ***Input Values for the Basel II Internal Ratings Based Approaches***

#### ***PD***

The value of a BSM bond may vary over its life, but it may only default at maturity. Under the maintained stochastic assumptions, the physical probability that a BSM bond defaults at maturity is,  $PD = \Phi(z_i^{df})$  where,

$$z_i^{df} = \left( \frac{\text{Log}(Par_i) - \text{Log}(A_{i0}) - \left( r_f + \lambda \sigma_M - \frac{\sigma_M^2 + \bar{\sigma}_i^2}{2} \right)}{\sqrt{\sigma_M^2 + \bar{\sigma}_i^2}} \right) \quad (27)$$

While Basel II regulations discuss data requirements for estimating a 1-year  $PD$  rate from historical data, there is no ambiguity about the definition of  $PD$  for the BSM bonds used in this analysis.

#### ***LGD and EAD***

The expected value of the bond's payoff at maturity, given that the bond defaults is,

$$E[\text{Min}(A_{i1}, Par_i) | A_{i1} < Par_i] = \frac{1}{\Phi(z_i^{df})} \int_{-\infty}^{z_i^{df}} A_{i0} e^{\left( r_f + \lambda \sigma_M - \frac{1}{2}(\sigma_M^2 + \sigma_i^2) \right) + \sqrt{(\sigma_M^2 + \sigma_i^2)} z} \phi(z) dz \quad (28)$$

Expression (28) can be used to define loss given default, but there is significant latitude as to how loss given default might be measured under Basel II. Recall that Basel II requires that  $LGD$  be measure relative to  $EAD$ , which must be at least as large as current gross exposure.

When  $LGD$  is calculated as a loss relative to a credit's current exposure (initial market value),  $LGD^{CE}$ , is,

$$LGD^{CE} = 1 - \frac{1}{B_{i0} \Phi(z_i^{df})} \int_{-\infty}^{z_i^{df}} A_{i0} e^{\left( r_f + \lambda \sigma_M - \frac{1}{2} (\sigma_M^2 + \sigma_i^2) \right) + \sqrt{(\sigma_M^2 + \sigma_i^2)} z} \phi(z) dz. \quad (29)$$

Expression (29) is a current exposure measure of calculating loss given default because it measures the default loss relative to the current exposure at the time that the capital allocation is being estimated.

Another potential way to measure loss given default using expression (28) is to measure  $LGD$  as the shortfall relative to the promised maturity value should a credit default. Define the future exposure measure of loss given default,  $LGD^{FE}$ , as loss is measured relative to the full contract value at the end of the capital allocation horizon,

$$LGD^{FE} = 1 - \frac{1}{Par_i \Phi(z_i^{df})} \int_{-\infty}^{z_i^{df}} A_{i0} e^{\left( r_f + \lambda \sigma_M - \frac{1}{2} (\sigma_M^2 + \sigma_i^2) \right) T + \sqrt{T(\sigma_M^2 + \sigma_i^2)} z} \phi(z) dz \quad (30)$$

Clearly  $LGD^{FE} > LGD^{CE}$ . Within the context of the Vasicek model and the Basel II framework, there is no theoretical preference for either measure of  $LGD$  as these models deal exclusively with the default process and are silent on the losses generated in default. Indeed there may be other methods for constructing  $LGD$  that are widely applied in practice. As the current exposure method for calculating  $LGD$  is clearly acceptable approach under Basel II, and since  $LGD^{CE}$  also produces the lowest minimum regulatory capital requirements, it is reasonable to assume that  $LGD^{CE}$  will be the preferred definition of AIRB banks.

### ***Alternative Minimum Capital Estimates***

The analysis includes 16 portfolios of one-year credits. The credit risk characteristics of the individual exposures are reported in Table 2. Individual credit  $PDs$  range from 23

basis points—for a bond with par values of 55, to 3.99 percent for a bond with a par value of 70. The  $LGD^{CE}$  characteristics (measured from initial market value) range from 1.40 percent to 3.28 percent. When  $LGD$  is measured on a future value basis,  $LGD^{FV}$  ranges from 6.22 to 8.34 percent. While the BSM model produces only modest  $LGD$ s relative to historical estimates of default losses on rated corporate bonds, the AIRB rule explicitly accounts for  $LGD$ , so *a priori*, there is no reason to expect that any specific set of  $LGD$  values may compromise the performance of the AIRB approach.

**Table 2: Credit Risk Characteristics of 1-Year Credits**

par value	initial market value	probability of default in percent	expected value given default	in percent		
				loss given default from initial value	loss given default from par value	yield to maturity
55	52.31	0.23	51.58	1.40	6.22	5.142
56	53.26	0.30	52.45	1.53	6.35	5.145
57	54.2	0.38	53.31	1.64	6.47	5.166
58	55.15	0.48	54.17	1.78	6.60	5.168
59	56.1	0.59	55.03	1.91	6.73	5.169
60	57.04	0.73	55.88	2.03	6.87	5.189
61	57.98	0.90	56.73	2.16	7.00	5.209
62	58.92	1.09	57.57	2.29	7.14	5.227
63	59.86	1.31	58.41	2.42	7.28	5.246
64	60.8	1.57	59.25	2.55	7.43	5.263
65	61.73	1.86	60.08	2.68	7.57	5.297
66	62.66	2.20	60.90	2.80	7.72	5.330
67	63.59	2.57	61.73	2.93	7.87	5.362
68	64.51	3.00	62.54	3.05	8.03	5.410
69	65.43	3.47	63.35	3.17	8.18	5.456
70	66.34	3.99	64.16	3.28	8.34	5.517

The results of the capital comparison are reported in Table 3 and plotted in Figure 1.

Depending on how  $EAD$  and  $LGD$  are defined, AIRB capital requirements may take on a

wide range of values. While the future exposure definition of *EAD* and *LGD* produce the largest AIRB capital measures, these capital assignments still fall short of the capital needed to achieve a 99.9 percent solvency rate for all but the safest credits analyzed. When capital requirements are calculated using the AIRB approach and the current exposure measure of *LGD*, the true capital needed to achieve the 99.9 percent target solvency rate may be more than 5 times larger than the minimum capital set by the AIRB approach.

**Table 3: Estimates of Portfolio Capital Requirements for 1-year Credits**

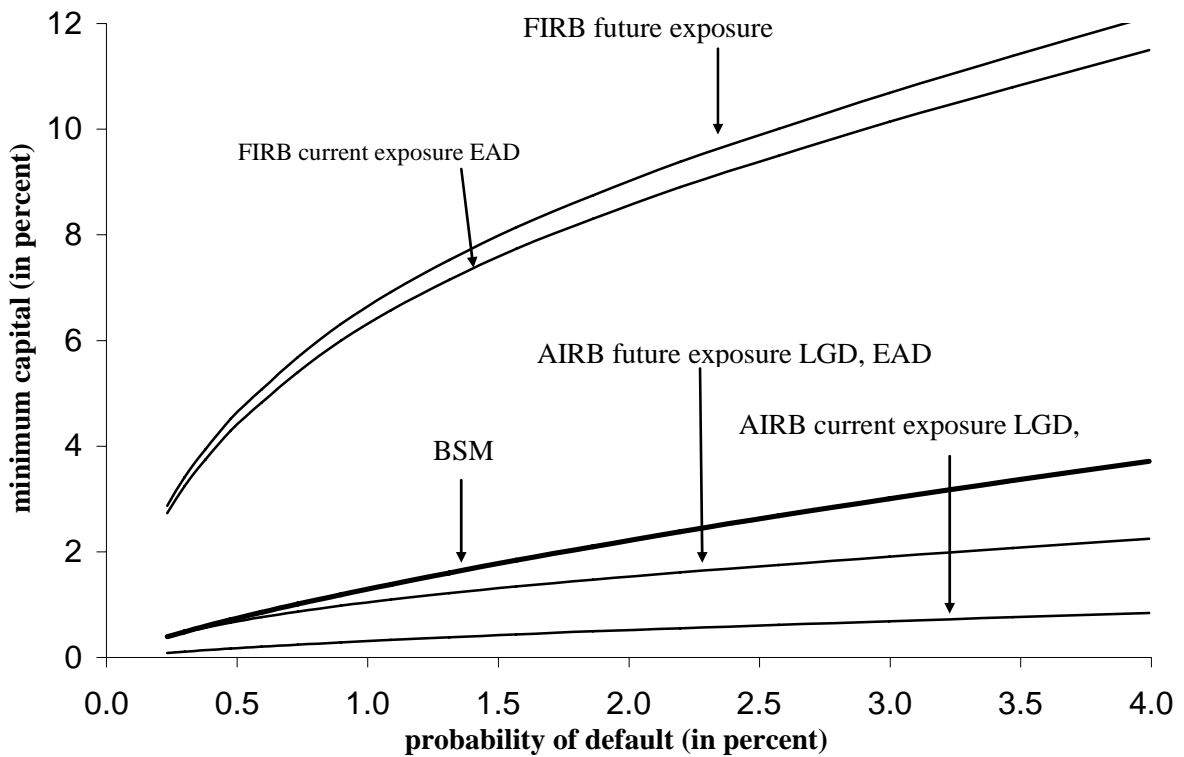
bond par value	probability of default in percent	99.9 percent BSM capital in percent	FIRB	FIRB	AIRB	AIRB
			capital current exposure EAD in percent	capital future exposure EAD in percent	capital current exposure LGD, EAD in percent	capital future exposure LGD,EAD in percent
55	0.233	0.396	2.733	2.874	0.086	0.400
56	0.298	0.487	3.241	3.407	0.110	0.479
57	0.379	0.593	3.747	3.941	0.136	0.565
58	0.476	0.715	4.297	4.519	0.169	0.660
59	0.593	0.854	4.822	5.072	0.206	0.761
60	0.732	1.011	5.400	5.680	0.245	0.869
61	0.896	1.187	5.999	6.312	0.287	0.980
62	1.088	1.384	6.573	6.917	0.334	1.097
63	1.311	1.601	7.146	7.521	0.384	1.217
64	1.568	1.839	7.733	8.140	0.439	1.343
65	1.862	2.098	8.307	8.747	0.494	1.472
66	2.196	2.379	8.906	9.381	0.554	1.608
67	2.574	2.681	9.499	10.009	0.619	1.751
68	2.997	3.005	10.138	10.687	0.687	1.906
69	3.469	3.348	10.797	11.386	0.761	2.069
70	3.992	3.712	11.497	12.131	0.839	2.249

Characteristics of individual bonds are reported in Table 2. The Basel FIRB and AIRB capital requirements include expected losses as well as unexpected losses.

As Figure 1 shows, owing to the 45 percent *LGD* assumption, the FIRB approach dramatically increases capital requirements relative to the AIRB approach. For credits with probabilities of default less than about 1.6 percent, FIRB capital requirements provide relief relative the 8 percent capital required by the 1988 Basel Accord. Notwithstanding capital

reductions for some credits, for the portfolios examined in this analysis, the FIRB will set capital requirements that are many times larger than are needed to achieve the regulatory target default rate of 0.1 percent. Under the FIRB approach, use of the future exposure *EAD* measure further compounds the capital surplus.

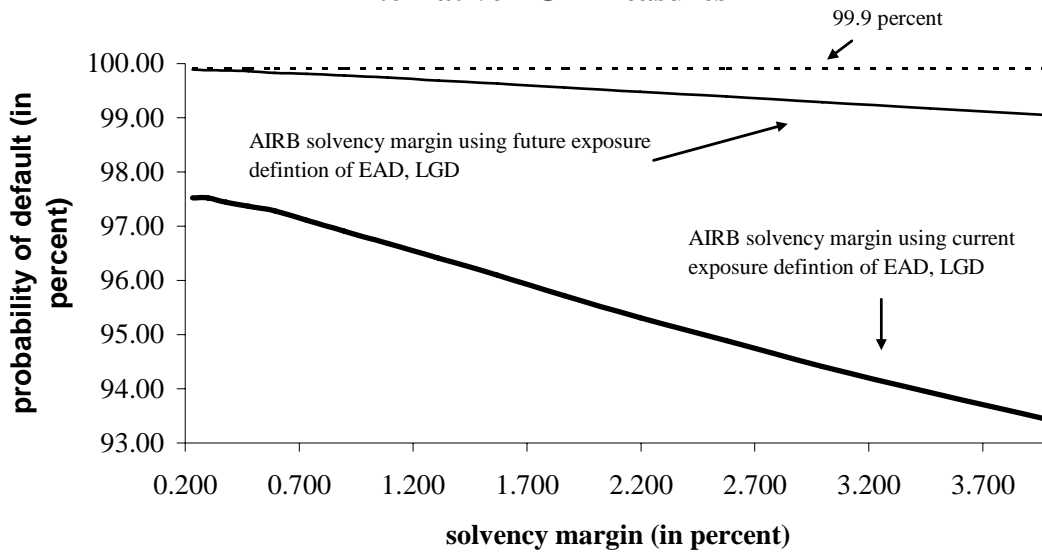
**Figure 1: Comparison of Alternative Minimum Capital Estimates for 1-Year Credits**



The BSM capital allocation rule can be inverted to recover the implied probability of default under the AIRB rule. Figure 2 plots the solvency margins that are set by the AIRB framework when capital is set using both the current exposure and the future exposures measures for *EAD* and *LGD*. The solvency margin set by the AIRB rule depends on the risk attributes of the credit and the

definitions used for the *EAD* and *LGD* inputs. For the credits examined in this analysis, using the current exposure measure of *EAD* and *LGD*, the AIRB solvency margin ranges from 97.5 percent to 93.4 percent. Under either measure of *EAD* and *LGD*, the AIRB solvency margin declines as credit risk increases.

**Figure 2: Approximate Solvency Margins Under Basel II and Alternative LGD Measures**



**Discussion**

No published study has attempted to measure the rigor of the prudential standard that will be set by Basel II capital regulations. Notwithstanding the BCBS's stated goal of creating a prudential standard consistent with a 99.9 percent bank solvency rate, the Basel II Quantitative Impact Studies (QIS) and subsequent IRB model calibration adjustments have not focused on producing calibrations consistent with any specific target solvency margin. Basel II deliberations have produced IRB model calibrations that create incentives to

promote AIRB adoption, but at the cost of significant diminution in the prudential standards that will apply to credit risk exposures in AIRB banks.

To gain perspective of the solvency standard set under the AIRB approach, consider for a moment that the AIRB approach default rates plotted in Figure 2 are approximately equal to the failure rate experienced by U.S. savings and loan institutions during the height of the 1980s S&L crisis. In 1988, the failure rate among insured savings and loans was 6.4 percent.<sup>18</sup> Over the 1980-1994 period, the annual compound average default rate of banks insured by the Federal Deposit Insurance Corporation was less than 1.2 percent, and even in the worst year (1988) of a period that has been characterized as a “banking crisis,” the default rate on FDIC insured banks never exceed 2 percent.<sup>19</sup>

The use of the BSM model as a benchmark of comparison merits discussion because the model has well-known empirical shortcomings. Econometric studies suggest that, on average, the BSM model overprices corporate bonds (i.e., underestimates required bond yields). Empirical evidence indicates that the BSM bias is related to maturity and credit quality.<sup>20</sup> BSM overpricing errors are more severe on short-term high quality credits. In the context of this capital calibration exercise, the observed pattern of bias implies that the BSM model analysis will likely understate the true amount of capital that is required to support a credit risky portfolio because the bank’s funding debt is likely to be overpriced by the BSM

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<sup>18</sup> “History of the Eighties—Lessons for the Future,” p. 168.

<sup>19</sup> See , “History of the Eighties—Lessons for the Future,” p. 479.

<sup>20</sup> See for example, Jones, Mason, and Rosenfeld (1984), Ogden (1987), or Eom, Helwege and Huang (2004).

framework. Recognizing the shortcomings of the BSM model, true economic capital allocations are likely larger than the estimates in this paper suggest, and true AIRB capital shortfalls are likely more severe than indicated.

To the extent that banks enjoy safety-net engendered subsidies that are attenuated by minimum regulatory capital requirements, the Base II IRB calibrations engender incentives that will encourage banking system assets to migrate toward AIRB banks. Asset migration could be achieved through consolidation or through an increase in the number banks that are granted regulatory approval for the AIRB approach. If regulatory hurdles and the fixed costs associated with adopting AIRB compatible systems are high, only the largest banks will favor the AIRB approach. Absent liberal regulatory approval policies or declines in AIRB compliant data and systems, strong economic incentives are in place to encourage industry consolidation into institutions that gain AIRB regulatory approval. Migration of assets into AIRB banks could substantially increase systemic risk in the financial system as even fully compliant AIRB banks may have high default rates unless market discipline of other regulations prohibit the realization of the full capital relief granted by the AIRB approach.<sup>21</sup>

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<sup>21</sup> For example, the leverage constraint under the U.S. system of prompt corrective action (12 U.S.C. Section 1831) may become the binding minimum regulatory capital requirement for many U.S. AIRB banks. See for example, the testimony of Donald E. Powell, Chairman Federal Deposit Insurance Corporation, on the Development of the New Basel Capital Accords before the Committee on Banking, Housing, and Urban Affairs. November 10, 2005, available at: <http://www.fdic.gov/news/news/speeches/archives/2005/chairman/spnov1005.html>

## 8. CONCLUSIONS

Compared to capital requirements calculated using a full equilibrium structural model of credit risk, the Basel II AIRB approach substantially understates the capital that is required to achieve the regulatory target of a 99.9 percent bank solvency rate. Estimates suggest that AIRB banks may have default rates in excess of 5 percent on their corporate sovereign and bank credit portfolios and still meet the minimum risk-based regulatory capital requirements promulgated by the June 2006 the AIRB approach. In contrast, the FIRB approach requires far more capital than is necessary to meet the regulatory target solvency standard.

The capital allocation analysis in this paper highlights important ambiguities in the Basel II framework and these ambiguities may lead to significant variation in the capital standards that apply across IRB banks. Current Basel II regulations are unnecessarily vague regarding the definitions of exposure at default and loss given default. Different interpretations of these concepts can lead to vastly different minimum regulatory capital requirements under the AIRB approach. Regardless of the definitions employed, the AIRB framework must still be recalibrated to produce an increase in minimum capital requirements if the 99.9 percent solvency margin target is to be respected. The recalibration of the AIRB rule must be linked to reformulated definitions of *EAD* and *LGD* that remove ambiguity and firmly establish the characteristics of the inputs used in the AIRB capital rule.

Under the current Basel II formulation, banks that adopt the AIRB approach will gain substantial regulatory capital relief without a commensurate reduction in their potential risk profile. FIRB banks, in contrast, will face a much stricter prudential standard. This dichotomy creates strong economic incentives for banking system assets to migrate into AIRB banks. Since the analysis suggests that AIRB banks potentially carry higher default

risk absent safety net support, the migration of banking system assets toward AIRB regulatory capital treatment is unlikely to enhance financial stability. Given the prudential weaknesses associated with the AIRB approach, the adoption of Basel II in its current form need not promote better risk management practices in banks or reduce systemic risk in the international banking system.

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